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# On Introduction of Fuzzy Dualistic Partial Metric Space

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***Abstract- In the present paper, we introduced a fuzzy partial metric space using the concepts of fuzzy metric space and partial metric space.***

***Keywords: Fuzzy point, Fuzzy metric space & Partial metric.***

## 1. Introduction

The concepts of fuzzy sets operations were first introduced by L.A.Zadeh in his classical paper [1] in the year 1965. Thereafter the paper of the C.L.Chang in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts [2]. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed.

## 2. Preliminaries

A fuzzy partial metric space is just a set X equipped with a function p of two variables which measures the distance between points. That is, distance function as defined as a partial metric p in [3], I just motivated it into fuzzy partial metric space.

A fuzzy set in X is called a fuzzy point $\Leftrightarrow$ if it takes the value 0 for any $y \in X$ except one; say $x \in X$ if its value at x is $\lambda$ $(0 < \lambda \leq 1)$ .the fuzzy point denoted by $O_x^{\lambda}$ where x is called its support.

**Definition 1:** A fuzzy point in fuzzy topological space (X, T) is a special fuzzy set with Membership function defined by

$O(x)=\begin{cases}1 & x=y\\ 0 & x\neq y\end{cases}$, where $0<\lambda<1$ is said to have support y, value $\lambda$ and is denoted by $o_y^{\lambda}$ or O(y, $\lambda$ ). It complement of the fuzzy point $o_y^{\lambda}$ is denoted by $o_y^{1-\lambda}$ and $o_y^{1}$ is called crops point.

**Definition 2:** A fuzzy partial metric on a nonempty set X is a function $p:X\times X\to[0,\infty)$ such that for all x, y, z $\in$ X: (P1)

$x=y\Leftrightarrow p(o_x^a,o_x^a)=p(o_x^a,o_y^b)=p(o_y^b,o_y^b)$ (P2)

$p(o_x^a,o_x^a)\le p(o_x^a,o_y^b)$ (P3)

$p(o_x^a,o_y^b)=p(o_y^b,o_x^a)$ (P4)

$p(o_x^a,o_y^b)\le p(o_x^a,o_z^c)+p(o_z^c,o_y^b)-p(o_z^c,o_z^c)$

A fuzzy partial metric space is a pair (X, p) such that X is a nonempty set and p is a fuzzy partial metric on X.

**Definition 3:** A fuzzy dualistic partial metric on a nonempty set X is a function $p:X\times X\to(-\infty,+\infty)$ such that for all x, y, z $\in$ X: (P1)

$x=y\Leftrightarrow p(o_x^a,o_x^a)=p(o_x^a,o_y^b)=p(o_y^b,o_y^b)$

(P2) $p(o_x^a,o_x^a)\le p(o_x^a,o_y^b)$

(P3) $p(o_x^a,o_y^b)=p(o_y^b,o_x^a)$

(P4)

$p(o_x^a,o_y^b)\le p(o_x^a,o_z^c)+p(o_z^c,o_y^b)-p(o_z^c,o_z^c)$

A fuzzy dualistic partial metric space is a pair (X, p) such that X is a nonempty set and p is a fuzzy partial metric on X.

**Definition 4:** A fuzzy weak partial metric on a nonempty set X is a function $p:X\times X\to[0,\infty)$ such that for all x, y, z $\in$ X:

(P1)

$x=y\Leftrightarrow p(o_x^a,o_x^a)=p(o_x^a,o_y^b)=p(o_y^b,o_y^b)$

(P2) $p(o_x^a,o_x^a)\le p(o_x^a,o_y^b)$

(P4)

$p(o_x^a,o_y^b)\le p(o_x^a,o_z^c)+p(o_z^c,o_y^b)-p(o_z^c,o_z^c)$

A fuzzy weak partial metric space is a pair (X, p) such that X is a nonempty set and p is a fuzzy weak partial metric on X.

**Example:** The pair $(R^{+},p)$ , where $p(o_x^a,o_y^b)=\mathrm{Max}\{x,y\}$ for all $x,y\in R^{+}$.

**Definition 3:** An open ball for a fuzzy partial metric $p:X\times X\to[0,\infty)$ is a set of the form, $B_\varepsilon^p(x)=\{y\in X/p(o_x^a,o_y^b)<\varepsilon\}$ for each $\varepsilon>0$, $x\in X$ .Note that, unlike their metric counterparts, some fuzzy partial metric open balls may be empty. For example, if $p(o_x^a,o_x^a)>0$, then $B^p_{p(o_x^a,o_x^a)}(x)=\phi$.

**Theorem 1:** The set of all open balls of a fuzzy partial metric $p:X\times X\to[0,\infty)$ is the basis of a fuzzy topology T[p] over X.

**Proof:**

As, $X=\bigcup_{x\in X}B^p_{p(o_x^a,o_x^a)+1}(x)$ and for any balls $B_\varepsilon^p(x)$ and $B_\delta^p(y)$

$B_\varepsilon^p(x)\cap B_\delta^o(y)=\cup\{B_\eta^p(z)/z\in B_\varepsilon^p(x)\cap B_\delta^p(y)\}$

Where,

$\eta=p(o_z^c,o_z^c)+\min\{\varepsilon-p(o_x^a,o_z^c),\delta-p(o_y^b,o_z^c)\}$

.

**Theorem 2:** For each fuzzy partial metric p, open ball $B_\varepsilon^p(a)$, and $x\in B_\varepsilon^p(a)$, there exists $\delta>0$ such that $x\in B_\delta^p(x)\subseteq B_\varepsilon^p(a)$.

**Proof:**

Suppose $x\in B_\varepsilon^p(a)\Rightarrow p(o_x^a,o_a^{a'})<\varepsilon$

Let

$$\delta=\varepsilon-p(o_x^a,o_a^{a'})+p(o_x^a,o_x^a)\Rightarrow\delta>0 \text{ as } \varepsilon>p(o_x^a,o_a^{a'})$$

Also

$$p(o_x^a,o_x^a)<\delta \quad \text{as} \quad \varepsilon>p(o_x^a,o_a^{a'})$$

Thus $x\in B_\delta^p(x)$

Suppose now that $y\in B_\delta^p(x)$

$$\therefore p(o_y^b,o_x^a)<\delta$$

$$\therefore p(o_y^b,o_x^a)<\varepsilon-p(o_x^a,o_a^{a'})+p(o_x^a,o_x^a)$$

$$\therefore p(o_y^b,o_x^a)+p(o_x^a,o_a^{a'})-p(o_x^a,o_x^a)<\varepsilon$$

$$\therefore p(o_y^b,o_x^a)<\varepsilon \quad \text{by (P4)}$$

$\therefore y\in B_\varepsilon^p(x)$

Thus

$B_\delta^p(x)\subseteq B_\varepsilon^p(a)$.

Using the last result it can be shown that each sequence $A_n\in X^\omega$ convergence to an object $a\in X$ if and only if $\lim_{n\to\infty} p(o_{A_n}^a,o_a^{a'})=p(o_a^{a'},o_a^{a'})$.

**Definition 4:** Fuzzy topological space $(X,T)$ is called a fuzzy $T_0$ space if and only if for any fuzzy points x and y such that $x\neq y$, either

$x\notin \bar{y}$ or $y\notin \bar{x}$

**Theorem 3:** Each fuzzy partial metric is fuzzy $T_0$.

**Proof:**

Suppose $p:X\times X\to[0,\infty)$ is fuzzy partial metric and suppose $x\neq y\in X$, then, from (P1) & (P2) which implies $p(o_x^a,o_x^a)\le p(o_x^a,o_y^b)$

and so $x\in B_\varepsilon^p(x)\wedge y\neq B_\varepsilon^p(x)$ where $\varepsilon=(p(o_x^a,o_x^a)+p(o_x^a,o_y^b))/2$.

So far we have shown that fuzzy partial metric p can quantify the amount of information in an object x using the numerical measure $p(o_x^a,o_x^a)$.

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